A unified multiscale vision of behavioral crowds

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On a Unified Multiscale Vision of Behavioral Crowds

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This paper proposes a multiscale vision to human crowds which provides a consistent description at the three possible modeling scales, namely, microscopic, mesoscopic, and macroscopic. The proposed approach moves from interactions at the microscopic scale and shows how the same modeling principles lead to kinetic and hydrodynamic models. Hence, a unified framework is developed which permits to derive models at each scale using the same principles and similar parameters. This approach can be used to simulate crowd dynamics in complex environments composed of interconnected areas, where the most appropriate scale of description can be selected for each area. This offers a pathway to the development of a multiscale computational model which has the capability to optimize the granularity of the description depending on the pedestrian local flow conditions. An important feature of the modeling at each scale is that the complex interaction between emotional states of walkers and their motion is taken into account.

Keywords: Crowd dynamics, multiscale vision, living systems, social dynamics.

AMS Subject Classification: 82D99, 91D10

1. Plan of the Paper

Mathematical modeling and numerical simulation of human crowds represent a challenging research field which has motivated an intense activity in recent years.

The interest is related not only to the theoretical value of this topic, but also to the potential benefits that these studies can bring to the society, for instance in the fire safety design process or in crowd management under specific threats, such as turmoil, panic and so forth [32].

Human crowds are complex systems, namely, systems composed of many entities undergoing nonlinearly additive interactions. Individual behaviors play the prominent role in the collective emerging behaviors. Therefore, the dynamics of a crowd cannot simply rely on mechanical and deterministic causality principles, but it should account for the heterogeneous behavior of individual entities, their emotional states, walking ability, and the the resulting changes that individual based interactions undergo. This is a specific feature of all living, hence complex, systems due to their ability to develop a self-organizing intelligence. In addition, collective learning ability [21] progressively modifies the rules of the interactions.

Indeed, researchers have effectively accepted the message delivered in [10], which addressed the modeling approach to account for behavioral features of walkers to be interpreted as active, rather than classical, particles. The rapidly growing interest of researchers towards active particle methods is witnessed in the edited book [9], where a broad bibliography is reported together with hints on modeling, qualitative analysis, and computational methods for differential models of living systems.

The interested reader is addressed to [35] for a survey of the literature produced in the past century on the physics and modeling of self-propelled particles, while the mathematical literature on crowd modeling by the individual-based and by the hydrodynamic approach has been reviewed more recently in [10]. The book [26] is mainly focused on the modeling at the macroscopic scale with some vision on multiscale problems. These references indicate that the description of the dynamics of the system by differential equations can be developed at the three usual scales, namely, microscopic (individual-based), macroscopic (hydrodynamic), and mesoscopic (kinetic).

Models at each scale present advantages and drawbacks. However, rather than discussing this topic, our paper chases the objective of developing a general unified approach related to a multiscale vision. This objective also accounts for the need of introducing aspects of social dynamics in large crowds. In more detail, the following two issues are taken into account:

Multiscale vision: By multiscale approach we mean selecting and modeling the microscopic dynamics which is necessary to correctly implement the derivation of mesoscopic and macroscopic models. In particular, the strategy tackles also the problem of the derivation of models at the macroscopic scale by suitable limits of kinetic models by letting tend to zero the distance between individuals.

Social behaviors: Recent papers have introduced the modeling of some aspects of social behaviors in crowds [6,13,16,51]. This development has been also motivated by human safety problems [33,41,43,47,48,52]. It has been shown that the strategy developed by walkers in stress conditions is subject to important modifications that might even induce unsafe situations [13]. The conceptual framework towards modeling social dynamics is delivered in [1].

Focusing on the multiscale vision, we observe that the modeling of individual-based interactions can take advantage of results reported in various papers which have been recently devoted to this topic [22,24,44,45]. Microscopic interaction models can be then used to derive kinetic-type models which, in turn under asymptotic limits, can lead to models at the macroscopic scale [4,17]. A hierarchy of phenomena at different scales is possible as shown in [28,29]. This approach has also been applied to vehicular traffic, see for instance [8].

Focusing on social behaviors, it is worth highlighting that models should have the capability of describing the dynamics of crowds composed of pedestrians whose emotional state is heterogeneously distributed and propagates in space and time. Examples range from the spreading of violence during a demonstration, where two groups of people confront each other, to the propagation of panic during emergency evacuations. Our paper aims at dealing with this challenging topic within the framework of the multiscale vision proposed in the following sections.

Bearing all of the above in mind, a description of the contents can be rapidly given as follows.

Section 2 firstly proposes some basic principles which should guide a common approach to model interactions at each scale. Then, some general mathematical structures, suitable to provide the conceptual framework towards modeling, are reported for each scale. We refer to structures already available in the literature which, however, need further modifications to chase the objective of our paper.

Section 3 focuses on crowds in unbounded domains and deals with the modeling of interactions, at each scale, according to the principles proposed in Section 2. These can be inserted into the aforementioned structures to derive models of collective behaviors.

The modeling of the dynamics in domains with boundaries and obstacles is studied in Section 4, which develops the approach in bounded domains to account for the presence of walls which are perceived by walkers at a distance from the boundary thus modifying their trajectories. This feature generates interesting analytic and computational problems with nonlocal boundary conditions.

Section 5 proposes a multiscale approach to model the propagation in space of specific behaviors, such as stress conditions, which can have an important influence on the support to safety problems, where stress conditions can drive the crowd towards irrational behaviors with influence on safety.

Finally, Section 6 proposes a critical analysis of the overall contents of this paper and looks at possible research perspectives focusing mainly on additional reasonings on multiscale problems referring to the derivation of models at the macroscopic scale from the underlying description delivered by the kinetic theory approach and on some perspective ideas on the modeling of swarms.

2. Modeling interactions and mathematical structures

Human crowds exhibit complexity features, typical of living systems, which can have an important impact on the collective dynamics. Indeed, unlike inert matter, the behavioral ability of heterogeneous human beings to develop walking strategies and to adapt themselves to the context generates observable effects arising from causes that often do not appear evident.

Collective behaviors emerging from interactions are the core of the complexity of crowd systems and significantly affect the individual behavioral strategy, which can be rational or irrational. As a matter of fact, even when the strategy is essentially rational, it may not be the best possible one, while emergent collective irrational behaviors can be generated under certain specific circumstances. In some extreme cases, as in stress situations due to incidents or overcrowding (see for instance [11]), interactions may generate results rather distant from any predictable outcome.

Therefore, the modeling approach, in addition to the aforementioned multiscale vision, should account for walkers having the *ability to express a strategy* which depends on the state of the entities in their surrounding environment. This ability is *heterogeneously distributed* and can include also different walking objectives. It also depends on the *quality of the environment*, namely, weather conditions, geometry of the venue, abrupt changes of directions, luminosity conditions, presence of smoke, and many others.

An additional aspect to be accounted for is the *nonlinearly additive* features of interactions as these involve immediate neighbors but also distant individuals. In some cases, the topological distribution of a fixed number of neighbors can play a prominent role in the development of strategies and interactions as living entities interact, in certain physical conditions, with a fixed number of other entities rather than with all those in their visibility domain.

This section defines at each scale the mathematical structures that can offer the conceptual framework for the derivation of specific models and subsequently presents the guidelines for the modeling of interactions. This presentation is confined to the case of a crowd where the emotional state is equally shared by all walkers. The structures refer to a crowd in unbounded domains, while the study of the role of obstacles and walls is treated in the next section. Hence, this section provides the conceptual framework for the derivation of models. The contents refer to the existing literature [10] which is critically analyzed and revisited.

2.1. *Variables and parameters of the modeling approach*

We consider the dynamics in a two dimensional domain Σ, where the crowd moves, while $\Sigma_0 \subseteq \Sigma$ denotes the region which contains the whole crowd at the initial time, $t = t_0$. The following reference quantities and parameters are introduced:

– ρ_M is the maximum density (occupancy) of walkers per square meter.

– ℓ is a characteristic length to be taken as the diameter of the circle containing Σ

or, if the motion is in unbounded domains, Σ_0.

– v_M is the highest individual speed which can be reached by a very fast walker in a free flow in high quality venues.

– $T = \ell/v_M$ is the characteristic time, corresponding to the time needed by a fast walker to cover the distance ℓ in a free flow in high quality venues.

– $\alpha \in [0,1]$ models the overall quality of the venue, where $\alpha = 0$ corresponds to very low quality which prevents motion, while $\alpha = 1$ to very high quality allowing fast motion.

– $\beta \in [0,1]$ models the overall stress of the crowd, where $\beta = 0$ corresponds to very low stress, while $\beta = 1$ to very high stress. The role of this parameter on the motion is defined focusing on interactions.

Let us now consider the variables to be used to represent, at each scale, the overall state of the system under consideration. As discussed below, all variables are made dimensionless with respect to characteristic quantities and take values in the range $[0,1]$.

Individual-based models - microscopic scale: The dependent variables are the positions $\boldsymbol{x}_i = \boldsymbol{x}_i(t) = (x_i(t), y_i(t))$ and the velocities $\boldsymbol{v}_i = \boldsymbol{v}_i(t) = (v_{x,i}(t), v_{y,i}(t))$, with $i \in \{1, \ldots, N\}$, of N walkers. Positions and velocities are referred to ℓ and v_M, respectively. The independent variable is the dimensionless time t, obtained by scaling the dimensional time by the characteristic time T.

Kinetic models - mesoscopic scale: The dependent variable is a probability distribution function $f = f(t, \boldsymbol{x}, \boldsymbol{v})$ at time t and position $\boldsymbol{x}$ over the *microscopic state* $\boldsymbol{v}$. The distribution function is normalized by ρ_M. The one-particle representation is used so that f is linked to the so-called test particle (walker) assumed to be representative of the whole system. Time, space, and the microscopic velocity are the independent variables.

Hydrodynamic models - macroscopic scale: The dependent variables are the local density $\rho = \rho(t, \boldsymbol{x})$ and the local mean velocity $\boldsymbol{\xi} = \boldsymbol{\xi}(t, \boldsymbol{x})$, where ρ is divided by ρ_M and the mean speed ξ is divided by v_M. The dimensionless time t and space $\boldsymbol{x}$ are the independent variables.

Visibility domain: The visibility domain has the same geometrical properties at each scale and is assumed to be an arc of circle symmetric with respect to the walker's velocity direction. At the *microscopic scale*, it is denoted by $\Omega(\boldsymbol{x}_i, \boldsymbol{\nu}_i)$, and it refers to the i-th walker in $\boldsymbol{x}_i$ walking with velocity $\boldsymbol{v}_i = v_i \boldsymbol{\nu}_i$, where v_i is the speed and $\boldsymbol{\nu}_i$ is the unit vector directed as $\boldsymbol{v}_i$; at the *mesoscopic scale*, it is denoted by $\Omega(\boldsymbol{x}, \boldsymbol{\nu})$ and it refers to the walker, called test walker, representative of the whole system, in $\boldsymbol{x}$ with velocity $\boldsymbol{v} = v\boldsymbol{\nu}$, where v is the speed and $\boldsymbol{\nu}$ is the unit vector directed as $\boldsymbol{v}$; at the *macroscopic scale*, it is denoted by $\Omega(\boldsymbol{x}, \boldsymbol{\nu}_{\boldsymbol{\xi}})$, and it refers to the elementary physical domain in $\boldsymbol{x}$ with locally averaged velocity $\boldsymbol{\xi} = \xi\, \boldsymbol{\nu}_{\boldsymbol{\xi}}$ where ξ is the mean speed and $\boldsymbol{\nu}_{\boldsymbol{\xi}}$ is the unit vector directed as the mean velocity.

2.2. *Mathematical structures*

The structures which provide the mathematical framework supporting the derivation of models at each scale are presented in this section. The reader interested to the pertinent bibliography is addressed to the survey [10].

Structures at the microscopic (individual based) scale: Let us consider the motion of N walkers in a two-dimensional domain, where N might depend on time, while N_0 is the number of individuals at the initial time. The dynamics of the system is defined by a large system of ordinary differential equations for the position and velocity of the walkers, considered as active particles:

$$\begin{cases} \dfrac{d\boldsymbol{x}_i}{dt} = \boldsymbol{v}_i, \\[2ex] \dfrac{d\boldsymbol{v}_i}{dt} = \mathbf{F}_i(\boldsymbol{x}_1, \dots, \boldsymbol{x}_N, \boldsymbol{v}_1, \dots, \boldsymbol{v}_N; \alpha, \beta) = \mathbf{F}_i(\boldsymbol{x}, \boldsymbol{v}; \alpha, \beta) \end{cases} \tag{2.1}$$

where $\mathbf{F}_i(\cdot)$ is a psycho-mechanical acceleration acting on the i–th walker based on the action of other walkers in his/her visibility domain $\Omega(\boldsymbol{x}_i, \boldsymbol{\nu}_i)$ which might be shaded by walls/obstacles, while $\boldsymbol{x}$ and $\boldsymbol{v}$ denote the whole set of positions and velocities. This term depends on the quality of the venue and on the emotional state, which can be modeled, respectively, by the parameters α and β. A simplification consists in modeling $\mathbf{F}_i$ as the superposition of binary interactions between pairs of walkers, but this assumption is questionable as interactions between walkers depend on the presence of all other walkers. The so-called *social force model* [36] is the reference model derived in the framework of (2.1).

More in general, the modeling of $\mathbf{F}_i$ might involve macroscopic quantities, e.g. the density, by a functional dependence to be properly defined for each specific model. In this case the following notation is used $\mathbf{F}_i = \mathbf{F}_i[\rho, \boldsymbol{\xi}](\boldsymbol{x}, \boldsymbol{v}; \alpha, \beta)$, while time might appear for non autonomous systems.

Structures at the mesoscopic (kinetic) scale: The representation is defined by the statistical distribution of the microscopic position and velocity of a *test walker*, given by the distribution function $f = f(t, \boldsymbol{x}, \boldsymbol{v})$. If f is locally integrable then $f(t, \boldsymbol{x}, \boldsymbol{v})\, d\boldsymbol{x}\, d\boldsymbol{v}$ is the infinitesimal number of pedestrians who, at time t, have a microscopic state $(\boldsymbol{x}, \boldsymbol{v})$ comprised in the elementary volume $d\boldsymbol{x}\, d\boldsymbol{v}$ of the phase space centered at $(\boldsymbol{x}, \boldsymbol{v})$.

Observable macroscopic quantities can be obtained, under suitable integrability assumptions, by moments of the distribution function. For instance, the dimensionless local density ρ and the total number of pedestrians N in Σ at time t are given, respectively, by

$$\rho(t, \boldsymbol{x}) = \int_{D_{\boldsymbol{v}}} f(t, \boldsymbol{x}, \boldsymbol{v})\, d\boldsymbol{v} \quad \text{and} \quad N(t) = \int_{\Sigma} \rho(t, \boldsymbol{x})\, d\boldsymbol{x}, \tag{2.2}$$

where $D_{\boldsymbol{v}} \subseteq \mathbb{R}^d$, being d the number of dimensions of the problem. Analogously, the mean velocity $\boldsymbol{\xi}$ and the speed variance σ can be computed as

$$\boldsymbol{\xi}(t,\, \boldsymbol{x}) = \frac{1}{\rho(t,\, \boldsymbol{x})} \int\limits_{D_{\boldsymbol{v}}} \boldsymbol{v} f(t,\, \boldsymbol{x},\, \boldsymbol{v})\, d\boldsymbol{v}, \tag{2.3}$$

and

$$\sigma(t,\, \boldsymbol{x}) = \frac{1}{\rho(t,\, \boldsymbol{x})} \int\limits_{D_{\boldsymbol{v}}} |\boldsymbol{v} - \boldsymbol{\xi}(t,\, \boldsymbol{x})|^2 f(t,\, \boldsymbol{x},\, \boldsymbol{v})\, d\boldsymbol{v}. \tag{2.4}$$

Kinetic models are stated in terms of an evolution equation for the distribution function f, deduced as a balance law in the space of the microscopic states. A basic structure is:

$$\begin{aligned} &\left(\frac{\partial}{\partial t} + \boldsymbol{v} \cdot \nabla_{\boldsymbol{x}}\right) f(t, \boldsymbol{x}, \boldsymbol{v}) \\ &\qquad = G[f](t, \boldsymbol{x}, \boldsymbol{v}; \alpha, \beta) - f(t, \boldsymbol{x}, \boldsymbol{v})\, L[f](t, \boldsymbol{x}, \boldsymbol{v}; \alpha, \beta), \end{aligned} \tag{2.5}$$

where $\nabla_{\boldsymbol{x}}$ denotes the gradient operator with respect to the space variables. In addition, G and L are operators acting on the distribution function f, which express the *gain* and the *loss* of pedestrians in the elementary volume of the phase space around the test microscopic state $(\boldsymbol{x},\, \boldsymbol{v})$.

The detailed expression of these terms corresponds to different ways of modeling pedestrian interactions at the microscopic scale; specializations of this structure have been proposed in [6] for models with discrete velocities and in [11] for models with continuous velocity distributions. Recent developments of this approach are proposed in [42]. The derivation can be obtained by distinguishing the interacting active particles into three types, namely, the *test*, the *field*, and the *candidate* particles. Their distribution functions are, respectively $f(t, \boldsymbol{x}, \boldsymbol{v})$, $f(t, \boldsymbol{x}^*, \boldsymbol{v}^*)$, and $f(t, \boldsymbol{x}, \boldsymbol{v}_*)$. The test particle is representative of the whole system, while the candidate particle can acquire, in probability, the micro-state of the test particle after interaction with the field particles. The test particle loses its state by interaction with the field particles.

In addition the following two quantities are introduced: The *interaction rate*

$$\eta[f](\boldsymbol{x}, \boldsymbol{x}^*, \boldsymbol{v}_*, \boldsymbol{v}^*; \alpha, \beta), \qquad \big(\text{resp.} \quad \eta[f](\boldsymbol{x}, \boldsymbol{x}^*, \boldsymbol{v}, \boldsymbol{v}^*; \alpha, \beta)\big)$$

which models the frequency by which a candidate (resp. test) particle in $\boldsymbol{x}$ interacts, in the visibility domain, with a field particle in $\boldsymbol{x}^*$, and the *transition probability density*

$$\mathcal{A}[f](\boldsymbol{v}_* \to \boldsymbol{v} | \boldsymbol{x}, \boldsymbol{x}^*, \boldsymbol{v}_*, \boldsymbol{v}^*; \alpha, \beta)$$

which models the probability density that a candidate particle in $\boldsymbol{x}$ modifies the velocity into that of the test particle due to the interaction with a field particle in the visibility domain. It is worth enlightening that square brackets have been used to denote that η and $\mathcal{A}$ can depend on f.

The following structure is formally derived:

$$\begin{aligned}
\left(\frac{\partial}{\partial t} + \boldsymbol{v}\cdot\nabla_{\boldsymbol{x}}\right) f(t,\boldsymbol{x},\boldsymbol{v}) &= J[f](t,\boldsymbol{x},\boldsymbol{v};\alpha,\beta) \\
&= \int_{D_{\boldsymbol{v}}\times D_{\boldsymbol{v}}} \int_{\Omega(\boldsymbol{x},\boldsymbol{\nu}_*)} \eta[f](\boldsymbol{x},\boldsymbol{x}^*,\boldsymbol{v}_*,\boldsymbol{v}^*;\alpha,\beta)\,\mathcal{A}[f](\boldsymbol{v}_*\to\boldsymbol{v}|\boldsymbol{x},\boldsymbol{x}^*,\boldsymbol{v}_*,\boldsymbol{v}^*;\alpha,\beta) \\
&\qquad\qquad \times f(t,\boldsymbol{x},\boldsymbol{v}_*)f(t,\boldsymbol{x}^*,\boldsymbol{v}^*)\,d\boldsymbol{x}^*\,d\boldsymbol{v}^*\,d\boldsymbol{v}_* \\
&\quad - f(t,\boldsymbol{x},\boldsymbol{v}) \int_{D_{\boldsymbol{v}}} \int_{\Omega(\boldsymbol{x},\boldsymbol{\nu})} \eta[f](\boldsymbol{x},\boldsymbol{x}^*,\boldsymbol{v},\boldsymbol{v}^*;\alpha,\beta)\, f(t,\boldsymbol{x}^*,\boldsymbol{v}^*)\,d\boldsymbol{x}^*\,d\boldsymbol{v}^*. \qquad (2.6)
\end{aligned}$$

Structures at the macroscopic (hydrodynamical) scale: The macroscopic Eulerian description can be adopted for large scale systems in which the local behavior of groups is sufficient to capture the global dynamics. Models at the macroscopic scale are mostly inspired by the equations of fluid dynamics. The approach of modeling crowd dynamics by modifying the equations of hydrodynamics has been arguably initiated by the pioneering paper [37], while a model which is even nowadays object of studies has been proposed in [39], where the equation of conservation of mass has been linked to a model suitable to describe the dynamics of the mean velocity by the assumption that walkers attempt to reduce density gradients. The interested reader is addressed to the book [26] which provides a general overview, critical analysis, and applications of crowd modeling by methods of continuum mechanics which are properly related at approaches at lower scales.

The structure of *second order* models is given by the balance equations for the mass and linear momentum:

$$\begin{cases}
\dfrac{\partial \rho}{\partial t} + \nabla_{\boldsymbol{x}}\cdot(\rho\,\boldsymbol{\xi}) = 0 \\[2ex]
\dfrac{\partial \boldsymbol{\xi}}{\partial t} + \boldsymbol{\xi}\cdot\nabla_{\boldsymbol{x}}\boldsymbol{\xi} = \mathcal{F}[\rho,\boldsymbol{\xi}](\boldsymbol{x};\alpha,\beta),
\end{cases} \qquad (2.7)$$

where $\mathcal{F}$ is the psycho-mechanical acceleration acting locally on pedestrians contained in an infinitesimal volume of the physical space. The key problem of the approach is precisely the modeling of $\mathcal{F}$ which might depend, in some functional form, on the density and the mean velocity.

2.3. *Rationale towards modeling interactions*

As mentioned, none of the representation and modeling scales presented in the previous section can't fully capture the complexity features of human crowds. In principle, the *microscopic scale* offers the most appropriate modeling framework, but leads to large systems of ordinary differential equations inducing nontrivial analytic and computational difficulties. These ultimately make the microscopic approach prone to fluctuations which affect the computation of macroscopic quantities from data at the microscopic scale. In addition, a small scale description requires a highly detailed modeling of individual behaviors, which may not entirely be phenomenologically observable.

Concerning the *mesoscopic scale*, we observe that the assumption of a continuous distribution of the microscopic states, borrowed from the kinetic theory of gases, is questionable in the present context, due to the typically much lower number of pedestrians in a crowd than molecules in a gas.

The *macroscopic scale* is susceptible to criticisms as well, since crowds clearly do not fit the paradigm of continuity of the matter. In addition, local averaging suppresses the heterogeneity which is a prominent feature to be taken into account. On the other hand, the really useful quantitative information that a model should deliver is required directly at the macroscopic scale. In fact, it is less prone to globally unnecessary details and to fluctuations, besides referring to quantities directly observable and measurable, such as mass density and flux, which depict well the emergence of collective patterns.

The common feature of all approaches is that the derivation of models relies on the mathematical description of interactions within the framework offered at each scale. The development of a multiscale approach requires that the modeling of interactions is based on the same principles at each scale. This requirement is a preliminary step for the derivation of macroscopic models from the underlying description at the microscopic scale, which might move from microscopic to mesoscopic by a common modeling of individual-based interactions, and, subsequently, from mesoscopic to macroscopic by asymptotic methods.

Bearing all of the above in mind, let us indicate the common features which should be taken into account, according to the authors' belief, in the modeling of interactions. The presentation is here given simply at a qualitative level, leaving their formalization to the next sections.

Note that, in the following, walkers will be generically referred to as *active particles*, (in short *a-particles*), having in mind a different meaning at each scale, namely, individuals at the micro-scale, statistical particles at the meso-scale, and number of individuals in the elementary physical space at the macroscopic scale.

The following five common features of particle interactions may be identified.

(1) All a-particles have a visibility angle related to their velocity direction and a visibility radius depending on the quality and shape of the venue, namely, the presence of obstacles or walls can reduce the area of the circular sector.
(2) All a-particles are subject to different stimuli, namely, a trend towards a well defined direction corresponding to a meeting point, a walking direction, the attraction by the motion of the other a-particles which, however, is contrasted by a desire to avoid overcrowded areas.
(3) The selection of the velocity direction corresponds to a weighted selection of the stimuli mentioned in Item 2 depending on the quality of the venue, the emotional state, and the local density.

(4) Once a walking direction has been selected, a-particles adapt their speed to local density depending also on the parameters the quality of the venue and pedestrians' emotional state.
(5) The presence of obstacles and walls geometrically modifies the visibility, and consequently the visibility domains, and induces an additional desire to avoid them.

The following remarks are in order:

Remark 2.1. A common feature of all structures is that interactions generate a *double nonlinearity* as equations not only present algebraic nonlinearities for the dependent variable, but also functional dependence on them which have been denoted by square brackets. This remark will be made precise at each scale of the modeling approach as we shall see in the next section.

Remark 2.2. The aforementioned five common features of interactions have been selected according to the authors' belief and experience. Additional study, supported by empirical data, might improve this selection. Validation should be developed as in [12], based on the ability of models to reproduce empirical data and depict emerging behaviors observed in experimental investigation.

Remark 2.3. In order to describe the space propagation of emotional states, β must be treated not as a constant parameter, but rather as a microscopic variable to be inserted in the interactions at each scale.

Remark 2.4. The multiscale vision presented in the next Sections 3–4 is limited, to avoid heavy notations, to the study of a crowd which shares the same strategy although heterogeneously distributed. However, introducing the interactions of different groups is necessary to model real flow conditions.

3. Derivation of models in unbounded domains

This section shows how the rationale presented in Section 2, focused on interactions by common modeling criteria, can be used to derive models at the three scales introduced in Subsection 2.2. The approach is developed for each scale in the next three subsections for a crowd flow in unbounded domains.

The models derived in the following refer to the mathematical structure proposed in Section 2. Note, however, that a further simplifying assumption is made in deriving kinetic-type models, namely, it is supposed that field particles trigger interactions but their microscopic state does not directly contribute neither to the interaction rate nor to the transition probability density. Some comments at the end of each subsection are deemed to enlighten the use of notations introduced in Remark 1.

3.1. *Derivation of individual-based models*

Let us consider the mathematical structure defined by Eq. (2.1), where the modeling essentially consists in deriving the acceleration term $\mathbf{F}_i$ for each a-particle i. We look for a model of the strategy by which each a-particle selects the velocity direction $\boldsymbol{\omega}_i$ and subsequently moves with an acceleration φ_i along $\boldsymbol{\omega}_i$. Polar coordinates are used for the velocity

$$\boldsymbol{v}_i = \{v_i, \theta_i\} = v_i\,(\cos\theta_i\,\boldsymbol{i} + \sin\theta_i\,\boldsymbol{j}) = v_i\boldsymbol{\nu}_i, \qquad \boldsymbol{\nu}_i = \frac{\boldsymbol{v}_i}{||\boldsymbol{v}_i||} = \cos\theta_i\,\boldsymbol{i} + \sin\theta_i\,\boldsymbol{j}, \tag{3.1}$$

where $\boldsymbol{i}$ and $\boldsymbol{j}$ are the unit vectors of an orthogonal frame, v_i is the speed and θ_i is the angle which identifies the velocity direction $\boldsymbol{\nu}_i$.

Let us now show how the rationale reported in Subsection 2.3 can provide the implementation of the structure (2.1), thus leading to a model suitable to describe the dynamics in unbounded domains.

(1) The *decision process* leading to the velocity dynamics assumes that an a-particle firstly selects the velocity direction $\boldsymbol{\omega}_i$ and subsequently modifies the speed. The selection of $\boldsymbol{\omega}_i$ is a weighted choice accounting for the direction towards the target $\boldsymbol{\nu}_i^{(t)}$, the attraction towards the main stream $\boldsymbol{\nu}_i^{(s)}$ of a-particles in Ω_i, and the search of paths with less congested local density $\boldsymbol{\nu}_i^{(v)}$.

(2) The *selection of the velocity direction* depends on the parameter β and is weighted by the local density ρ_i. In more details, increasing values of β correspond to a trend towards the stream with respect to the trend towards the target, while the local density increases the trend towards vacuum zones.

Detailed calculations, corresponding to the qualitative behaviors conjectured in Items 1–2, yield:

$$\boldsymbol{\omega}_i = \boldsymbol{\omega}_i[\rho, \boldsymbol{\xi}](\boldsymbol{x}_i, \boldsymbol{\nu}_i; \beta) = \frac{\rho_{\boldsymbol{\nu}_i}\,\boldsymbol{\nu}_i^{(v)} + \big(1 - \rho_{\boldsymbol{\nu}_i}\big)\dfrac{\beta\,\boldsymbol{\nu}_i^{(s)} + (1-\beta)\boldsymbol{\nu}_i^{(t)}}{||\beta\,\boldsymbol{\nu}^{(s)} + (1-\beta)\boldsymbol{\nu}_i^{(t)}||}}{\left\|\rho_{\boldsymbol{\nu}_i}\,\boldsymbol{\nu}_i^{(v)} + \big(1 - \rho_{\boldsymbol{\nu}_i}\big)\dfrac{\beta\,\boldsymbol{\nu}_i^{(s)} + (1-\beta)\boldsymbol{\nu}_i^{(t)}}{||\beta\,\boldsymbol{\nu}_i^{(s)} + (1-\beta)\boldsymbol{\nu}_i^{(t)}||}\right\|}, \tag{3.2}$$

where $\boldsymbol{\nu}_i$ has been defined in (3.1), while the direction towards empty zones, the attraction towards the main stream, and the direction towards the target are denoted, respectively, by the unit vectors

$$\boldsymbol{\nu}_i^{(v)} = -\frac{\nabla_{\boldsymbol{x}}\rho_{\boldsymbol{\nu}_i}}{\|\nabla_{\boldsymbol{x}}\rho_{\boldsymbol{\nu}_i}\|}, \qquad \boldsymbol{\nu}_i^{(s)} = \frac{\boldsymbol{\xi}_{\boldsymbol{\nu}_i}}{\|\boldsymbol{\xi}_{\boldsymbol{\nu}_i}\|}, \qquad \text{and} \qquad \boldsymbol{\nu}_i^{(t)} = \frac{\boldsymbol{x}_t - \boldsymbol{x}_i}{\|\boldsymbol{x}_t - \boldsymbol{x}_i\|}, \tag{3.3}$$

being x_t the target point. In Eqs. (3.2) and (3.3), $\rho_{\boldsymbol{\nu}_i}$ and $\boldsymbol{\xi}_{\boldsymbol{\nu}_i}$ are given by:

$$\rho_{\boldsymbol{\nu}_i} = \frac{1}{measure(\Omega(\boldsymbol{x}_i, \boldsymbol{\nu}_i))}\sum_{j\in\Omega}\delta(\boldsymbol{x}_i - \boldsymbol{x}_j), \tag{3.4}$$

$$\boldsymbol{\xi}_{\boldsymbol{\nu}_i} = \frac{1}{\rho_{\boldsymbol{\nu}_i}}\sum_{j\in\Omega}\boldsymbol{v}_j\delta(\boldsymbol{v}_i - \boldsymbol{v}_j)\delta(\boldsymbol{x}_i - \boldsymbol{x}_j). \tag{3.5}$$

where $\Omega = \Omega(\boldsymbol{x}_i, \boldsymbol{\nu}_i)$. Note that the definition of the unit vectors given by Eq. (3.3) suppose that suitable regularity conditions hold for the density and velocity field averaged over the visibility domain, given respectively by Eqs. (3.4) and (3.5).

The *dynamics of the speed* depends on the parameter α which enhances modifications of the speed and on the difference between the densities over the visibility domains $\Omega(\boldsymbol{x}_i, \boldsymbol{\nu}_i)$ and $\Omega(\boldsymbol{x}_i, \boldsymbol{\omega}_i)$ corresponding to directions $\boldsymbol{\nu}_i$ and $\boldsymbol{\omega}_i$, respectively:

$$\varphi_i = \varphi_i[\rho](\boldsymbol{x}_i, \boldsymbol{v}_i, \boldsymbol{\omega}_i; \alpha) = \begin{cases} \alpha\,(1 - v_i)(\rho_{\boldsymbol{\nu}_i} - \rho_{\boldsymbol{\omega}_i}), & \rho_{\boldsymbol{\nu}_i} \geq \rho_{\boldsymbol{\omega}_i} \\ \\ \alpha\, v_i(\rho_{\boldsymbol{\nu}_i} - \rho_{\boldsymbol{\omega}_i}), & \rho_{\boldsymbol{\nu}_i} < \rho_{\boldsymbol{\omega}_i}, \end{cases} \tag{3.6}$$

where $\rho_{\boldsymbol{\omega}_i}$ is the density computed in the visibility domain related to $\boldsymbol{\omega}_i$.

The acceleration term is then obtained as follows:

$$\mathbf{F}_i = \mathbf{F}_i[\rho, \boldsymbol{\xi}](\boldsymbol{x}_i, \boldsymbol{v}_i; \alpha, \beta) = \varphi_i[\rho](\boldsymbol{x}_i, \boldsymbol{v}_i, \boldsymbol{\omega}_i; \alpha)\, \boldsymbol{\omega}_i[\rho, \boldsymbol{\xi}](\boldsymbol{x}_i, \boldsymbol{\nu}_i; \beta), \tag{3.7}$$

where $\boldsymbol{\omega}_i$ and φ_i are delivered by Eqs. (3.2) and (3.6). Let us highlight that the acceleration $\mathbf{F}_i$ is a nonlocal quantity which depends on the averaged state of all a-particles in the domains $\Omega(\boldsymbol{x}_i, \boldsymbol{\nu}_i)$ and $\Omega(\boldsymbol{x}_i, \boldsymbol{\omega}_i)$.

Remark 3.1. The functional dependence of $\boldsymbol{\omega}_i$ on ρ, and $\boldsymbol{\xi}$ in Eq. (3.2) is related to the fact that ξ defines the visibility domain Ω_i for each particle as well as the stream direction, while ρ leads to all subsequent calculations. This dependence is explicitly indicated, to avoid heavy notations, only on the left-hand side of the equations, but it is implicit in the right-hand side equations. This explains the square brackets in Eq. (3.7).

Remark 3.2. The role of the parameters is that α contributes to the acceleration, while β to the selection of the direction. In more detail, increasing values of α, i.e., the quality of the venue, induce increasing values of the acceleration, while increasing values of β, i.e., the intensity of the stress, induce increasing values of the attraction towards the main stream.

3.2. *Derivation of kinetic-type models*

The derivation of kinetic models moves from the structure defined by Eq. (2.6) and is carried out by extending the rationale proposed at the microscopic scale to model the terms η and $\mathcal{A}$. As in the microscopic case, it is convenient to introduce polar coordinates, namely, the velocity $\boldsymbol{v} = \{v, \boldsymbol{\nu}\}$ in a plane motion is given by the speed v and the velocity direction $\boldsymbol{\nu}$. Accordingly, the transition probability density $\mathcal{A}$ which describes the dynamics of the velocity, namely speed and direction, can be formally written as $\mathcal{A}[f](v_* \to v, \boldsymbol{\nu}_* \to \boldsymbol{\nu}; \alpha, \beta)$ depending on the parameters involved in a detailed modeling of interactions.

A simple way to model the encounter rate consists in assuming that it is constant, namely, $\eta = \eta_0$, and assuming that the transition probability density does not depend on position and velocity of the field particle.

Substituting η into (2.6) yields:

$$\left(\frac{\partial}{\partial t} + \boldsymbol{v}\cdot\nabla_{\boldsymbol{x}}\right) f(t,\boldsymbol{x},\boldsymbol{v}) = J[f](t,\boldsymbol{x},\boldsymbol{v};\alpha,\beta)$$

$$= \eta_0 \int_{D_{\boldsymbol{v}}} \mathcal{A}[f](v_* \to v,\, \boldsymbol{\nu}_* \to \boldsymbol{\nu}|\boldsymbol{x},\boldsymbol{v}_*;\alpha,\beta) f(t,\boldsymbol{x},\boldsymbol{v}_*)\rho_{\boldsymbol{\nu}_*}(\boldsymbol{x},\boldsymbol{\nu}_*)\, d\boldsymbol{v}_*$$

$$- \eta_0\, \rho_{\boldsymbol{\nu}}(t,\boldsymbol{x},\boldsymbol{\nu}) f(t,\boldsymbol{x},\boldsymbol{v}). \tag{3.8}$$

where $\rho(t,\boldsymbol{x},\boldsymbol{\nu}_*)$ and $\rho(t,\boldsymbol{x},\boldsymbol{\nu})$ are the densities in the visibility domain related to the velocity directions $\boldsymbol{\nu}_*$ and $\boldsymbol{\nu}_*$, respectively, i.e.,

$$\rho_{\boldsymbol{\nu}_*} = \rho_{\boldsymbol{\nu}_*}[f](t,\boldsymbol{x},\boldsymbol{\nu}_*) = \int_{D_{\boldsymbol{v}}} \int_{\Omega(\boldsymbol{x},\boldsymbol{\nu}_*)} f(t,\boldsymbol{x}^*,\boldsymbol{v}^*) d\boldsymbol{x}^* d\boldsymbol{v}^* \tag{3.9}$$

$$\rho_{\boldsymbol{\nu}} = \rho_{\boldsymbol{\nu}}[f](t,\boldsymbol{x},\boldsymbol{\nu}) = \int_{D_{\boldsymbol{v}}} \int_{\Omega(\boldsymbol{x},\boldsymbol{\nu})} f(t,\boldsymbol{x}^*,\boldsymbol{v}^*) d\boldsymbol{x}^* d\boldsymbol{v}^* \tag{3.10}$$

The modeling of $\mathcal{A}$ can be developed following the same rationale proposed at the microscopic scale, namely, *the a-particle firstly modifies the velocity direction and subsequently the speed.* A simple model consists in assuming that this process is described by a product of delta functions as follows:

$$\mathcal{A}[\rho,\boldsymbol{\xi}](v_* \to v,\, \boldsymbol{\nu}_* \to \boldsymbol{\nu}|\boldsymbol{x},\boldsymbol{v}_*;\alpha,\beta) =$$
$$\delta\big(v - \varphi_*[\rho](\boldsymbol{x},\boldsymbol{v}_*,\boldsymbol{\omega}_*;\alpha)\big)\, \delta\big(\boldsymbol{\nu} - \boldsymbol{\omega}_*[\rho,\boldsymbol{\xi}](\boldsymbol{x},\boldsymbol{\nu}_*;\beta)\big), \tag{3.11}$$

where, in analogy with the microscopic approach, $\boldsymbol{\omega}_*$ and φ_* are, respectively, given by:

$$\boldsymbol{\omega}_* = \boldsymbol{\omega}_*[\rho,\boldsymbol{\xi}](\boldsymbol{x},\boldsymbol{\nu}_*;\beta) = \frac{\rho_{\boldsymbol{\nu}_*}\, \boldsymbol{\nu}_*^{(v)} + (1-\rho_{\boldsymbol{\nu}_*}) \dfrac{\beta\, \boldsymbol{\nu}_*^{(s)} + (1-\beta)\boldsymbol{\nu}_*^{(t)}}{||\beta\, \boldsymbol{\nu}_*^{(s)} + (1-\beta)\boldsymbol{\nu}_*^{(t)}||}}{\left\|\rho_{\boldsymbol{\nu}_*}\, \boldsymbol{\nu}_*^{(v)} + (1-\rho_{\boldsymbol{\nu}_*}) \dfrac{\beta\, \boldsymbol{\nu}_*^{(s)} + (1-\beta)\boldsymbol{\nu}_*^{(t)}}{||\beta\, \boldsymbol{\nu}_*^{(s)} + (1-\beta)\boldsymbol{\nu}_*^{(t)}||}\right\|}, \tag{3.12}$$

where

$$\boldsymbol{\nu}_*^{(v)} = -\frac{\nabla_{\boldsymbol{x}}\rho_{\boldsymbol{\nu}_*}}{\|\nabla_{\boldsymbol{x}}\rho_{\boldsymbol{\nu}_*}\|}, \qquad \boldsymbol{\nu}_*^{(s)} = \frac{\boldsymbol{\xi}_{\boldsymbol{\nu}_*}}{\|\boldsymbol{\xi}_{\boldsymbol{\nu}_*}\|}, \qquad \text{and} \qquad \boldsymbol{\nu}_*^{(t)} = \frac{\boldsymbol{x}_t - \boldsymbol{x}_*}{\|\boldsymbol{x}_t - \boldsymbol{x}_*\|}, \tag{3.13}$$

being $\boldsymbol{\xi}_{\boldsymbol{\nu}_*}$ the mean velocity in the visibility domain, while

$$\varphi_* = \varphi_*[\rho](\boldsymbol{x},\boldsymbol{v}_*,\boldsymbol{\omega}_*;\alpha) = \begin{cases} v_* + \alpha\,(1-v_*)(\rho_{\boldsymbol{\nu}_*} - \rho_{\boldsymbol{\omega}_*}), & \rho_{\boldsymbol{\nu}_*} \geq \rho_{\boldsymbol{\omega}_*}, \\ \\ v_* + \alpha\, v_*(\rho_{\boldsymbol{\nu}_*} - \rho_{\boldsymbol{\omega}_*}), & \rho_{\boldsymbol{\nu}_*} < \rho_{\boldsymbol{\omega}_*}. \end{cases} \tag{3.14}$$

Here φ_* refer to the instantaneous stochastic modification of the speed and not precisely to the acceleration.

Remark 3.3. The selection of the velocity direction and the subsequent modification of the speed are modeled by a rationale analogous to that applied in the modeling at the micro-scale. This functional dependence is put in square brackets in the transition probability density $\mathcal{A}[f](v_* \to v,\, \boldsymbol{\nu}_* \to \boldsymbol{\nu}|\boldsymbol{x}, \boldsymbol{v}_*; \alpha, \beta)$. Analogous dependence, in this case for the local density, might be discovered if the assumption of constant interaction rate is replaced by assuming that η grows with ρ starting from a sentinel level.

3.3. *Derivation of models at the macro-scale*

The derivation of hydrodynamical models moves from the structure defined by Eq. (2.7). Models can be obtained by modeling the acceleration term $\mathcal{F}$ based on the same rationale proposed at the microscopic scale. Namely, a-particles in the elementary volume $d\boldsymbol{x}$ first select a direction $\boldsymbol{\omega}$ and subsequently accelerate or decelerate according to the local density conditions. Hence, the same rationale of the previous subsections yields:

$$\mathcal{F} = \mathcal{F}[\rho, \boldsymbol{\xi}](\boldsymbol{x}; \alpha, \beta) = \varphi[\rho](\boldsymbol{x}, \boldsymbol{\xi}; \alpha)\, \boldsymbol{\omega}[\rho, \boldsymbol{\xi}](\boldsymbol{x}; \beta), \tag{3.15}$$

where $\boldsymbol{\omega}$ is computed as in Eq. (3.12),

$$\boldsymbol{\omega} = \boldsymbol{\omega}[\rho, \boldsymbol{\xi}](\boldsymbol{x}; \beta) = \frac{\rho_{\boldsymbol{\xi}}\, \boldsymbol{\nu}^{(v)} + (1-\rho_{\boldsymbol{\xi}}) \dfrac{\beta\, \boldsymbol{\nu}^{(s)} + (1-\beta)\boldsymbol{\nu}^{(t)}}{||\beta\, \boldsymbol{\nu}^{(s)} + (1-\beta)\boldsymbol{\nu}^{(t)}||}}{\left|\left|\rho_{\boldsymbol{\xi}}\, \boldsymbol{\nu}^{(v)} + (1-\rho_{\boldsymbol{\xi}}) \dfrac{\beta\, \boldsymbol{\nu}^{(s)} + (1-\beta)\boldsymbol{\nu}^{(t)}}{||\beta\, \boldsymbol{\nu}^{(s)} + (1-\beta)\boldsymbol{\nu}^{(t)}||}\right|\right|}, \tag{3.16}$$

and

$$\varphi = \varphi[\rho, \boldsymbol{\xi}](\boldsymbol{x}, \boldsymbol{\omega}; \alpha) = \begin{cases} \alpha\,(1-\xi)(\rho_{\boldsymbol{\xi}} - \rho_{\boldsymbol{\omega}}), & \rho_{\boldsymbol{\xi}} \geq \rho_{\boldsymbol{\omega}}, \\[2ex] \alpha\,\xi(\rho_{\boldsymbol{\xi}} - \rho_{\boldsymbol{\omega}}), & \rho_{\boldsymbol{\xi}} < \rho_{\boldsymbol{\omega}}. \end{cases} \tag{3.17}$$

Remark 3.4. The selection of the velocity direction and the subsequent modification of the speed are modeled by a rationale analogous to the applied in the modeling at the lower scales. This naturally implies that the acceleration term $\mathcal{F}$ depends on the density and mean speed. Here, square brackets are used to denote functional dependence, while the role of the parameters α and β is analogous to that at the low scale.

4. Modeling flows in the presence of obstacles and walls

The modeling approach presented in the previous subsections is valid for crowds in unbounded domains. Herein, we instead consider the modeling of the dynamics in venues which include obstacles, walls, and exits.

The initial value problem for models in unbounded domain, which we have studied in Section 3, can be transferred to an initial-boundary value problem. However, it is not simply a matter of implementing boundary conditions, but also of modifying the models to account for the sensitivity of a-particles to the presence of walls. A deep analysis of this key problem has been proposed in [3]. Such a sensitivity modifies the walking strategy and hence the trajectories. Following the rationale proposed in the previous subsections, the models should include modified velocity direction and acceleration terms. In addition, boundary conditions are required since walkers, in probability, might reach the boundaries even though their walking strategy encompasses the tendency to keep distance from them.

The modeling of the velocity direction is achieved by introducing the distance $\gamma = \gamma(\boldsymbol{x}, \boldsymbol{\nu})$ between the localization of a-particles and the wall, measured along the velocity direction. The role of γ is such that, when $\gamma \to 0$, the attraction to the exit becomes dominant with respect to the trends to vacuum and stream which, in turn, tend to zero. Therefore, γ plays the role of a weight for the trend to avoid walls. Once the new direction has been chosen, then the acceleration term is modeled exactly as in the preceding subsections. It is worth mentioning that the use of dimensionless space coordinates implies that $\gamma \in [0, 1]$ and that $\gamma = 0$ if the velocity direction happens to be precisely addressed to the exit. This rationale should be specialized at each scale by an appropriate calculation of γ. The modeling is achieved by implementing a dynamics by which a-particles cannot penetrate into walls.

This section is organized in two parts: Firstly, the derivation of models and of the statement of boundary conditions is developed at each scale, and, subsequently, a description of the overall rationale towards the derivation of models accounting also for the presence of walls is presented.

4.1. *Derivation of models and boundary conditions*

In the following, the new velocity direction is derived by the rationale which has been defined above accounting, in addition, for the role of γ. Full details are given for models at the microscopic scale, while only the technical differences are presented for the mesoscopic and macroscopic modeling approaches.

Microscopic scale: The velocity direction modeling of $\boldsymbol{\omega}_i^B$, corresponding to each a-particle, is obtained from $\boldsymbol{\omega}_i^F$, the flow direction selected by each particle in unbounded domains, and from $\boldsymbol{\nu}_B^{(t)}$, the target direction at the point $\boldsymbol{x}_B \in \partial\Sigma$, identified by the intersection of the velocity direction of the a-particle in $\boldsymbol{x}_i$ with the wall. Therefore, the weight γ can be applied to both directions as follows:

$$\boldsymbol{\omega}_i^B = \boldsymbol{\omega}_i^B[\rho, \boldsymbol{\xi}](\boldsymbol{x}_i, \boldsymbol{\nu}_i; \beta) = \frac{\gamma(\boldsymbol{x}_i, \boldsymbol{\nu}_i)\, \boldsymbol{\omega}_i^F[\rho, \boldsymbol{\xi}](\boldsymbol{x}_i, \boldsymbol{\nu}_i; \beta) + \big(1 - \gamma(\boldsymbol{x}_i, \boldsymbol{\nu}_i)\big)\, \boldsymbol{\nu}_B^{(t)}(\boldsymbol{x}_i)}{\|\gamma(\boldsymbol{x}_i, \boldsymbol{\nu}_i)\, \boldsymbol{\omega}_i^F[\rho, \boldsymbol{\xi}](t, \boldsymbol{x}_i, \boldsymbol{\nu}_i; \beta) + \big(1 - \gamma(\boldsymbol{x}_i, \boldsymbol{\nu}_i)\big)\, \boldsymbol{\nu}_B^{(t)}(\boldsymbol{x}_i)\|}. \tag{4.1}$$

Subsequently, the acceleration term can be computed as in (3.6-3.7), however accounting for the velocity direction computed by (4.1).

Mesoscopic scale: The same reasonings can be applied to the modeling of the dynamics at the mesoscopic scale where, in addition, boundary conditions for kinetic models have to be implemented.

Focusing on the derivation of the model, the same structure defined in (3.8) can be used, but the calculation of the velocity direction and the speed have to account for (4.1) referred to the test particle. The modeling of the transition probability density is developed as in Subsection 3.2 accounting for $\boldsymbol{\omega}^B$ and consequently for the post-interaction velocities.

Concerning the statement of the boundary conditions, the difference with respect to the microscopic scale consists in the statistical description of the flow, and an appropriate scattering model needs to be given. In more detail, we assume that the interaction with the wall at $\boldsymbol{x}_B \in \partial\Sigma$ modifies the velocity according to the following statistical boundary conditions which impose zero-net-flux at the solid surface:

$$f(t, \boldsymbol{x}_B, \boldsymbol{v})|\boldsymbol{v} \cdot \boldsymbol{n}_B| = \int_{\boldsymbol{v}_* \cdot \boldsymbol{n}} \delta\big(v - \varphi_*[\rho](\boldsymbol{x}_B, \boldsymbol{v}_*, \boldsymbol{\nu}_B^{(t)}; \alpha)\big)\, \delta\big(\boldsymbol{\nu} - \boldsymbol{\nu}_B^{(t)}\big) f(t, \boldsymbol{x}_B, \boldsymbol{v}_*)|\boldsymbol{v}_* \cdot \boldsymbol{n}_B| dv_*, \tag{4.2}$$

where $\boldsymbol{n}_B$ is the unit vector orthogonal to the wall at $\boldsymbol{x}_B$ and φ_* is the speed as given by Eq. (3.14).

Macroscopic scale: The mathematical structure is that of (3.15), but the following velocity direction has to be used to account for the influence of walls over the velocity direction:

$$\boldsymbol{\omega}^B = \boldsymbol{\omega}^B[\rho, \boldsymbol{\xi}](\boldsymbol{x}, \boldsymbol{\nu}_{\boldsymbol{\xi}}; \beta) = \frac{\gamma(\boldsymbol{x}, \boldsymbol{\nu}_{\boldsymbol{\xi}})\, \boldsymbol{\omega}^F[\rho, \boldsymbol{\xi}](t, \boldsymbol{x}, \boldsymbol{\nu}_{\boldsymbol{\xi}}; \beta) + \big(1 - \gamma(\boldsymbol{x}, \boldsymbol{\nu}_{\boldsymbol{\xi}})\big)\, \boldsymbol{\nu}_B^{(t)}(\boldsymbol{x})}{\|\gamma(\boldsymbol{x}, \boldsymbol{\nu}_{\boldsymbol{\xi}})\, \boldsymbol{\omega}^F[\rho, \boldsymbol{\xi}](t, \boldsymbol{x}, \boldsymbol{\nu}_{\boldsymbol{\xi}}; \beta) + \big(1 - \gamma(\boldsymbol{x}, \boldsymbol{\nu}_{\boldsymbol{\xi}})\big)\, \boldsymbol{\nu}_B^{(t)}(\boldsymbol{x})\|}, \tag{4.3}$$

while the acceleration terms can be computed as in (3.17), but accounting for (4.3) which modifies the modulus of the acceleration.

4.2. *Additional reasonings on the modeling of interactions*

This subsection provides a final summary of the strategy to derive models at different scales, always according to the same rationale. A critical analysis follows with the aim of contributing to further modeling hints. The decision process by which walkers modify their dynamics can be summarized as follows:

(1) **Hierarchy:** Selection of the velocity direction and subsequent modification of the speed.

(2) **Hints towards the selection of the velocity directions:** Walkers are subject to the following trends: Reaching the nearest exit, avoiding overcrowded

areas, attraction towards the main steam, avoiding walls.

(3) **Selection of the velocity direction:** The hierarchy of the selection of the velocity direction is as follows: Walkers first select between target and stream directions according to their stress conditions; subsequently, they choose between this firstly selected direction and the trend towards less congested areas; finally they modify this direction to avoid interactions with walls.

(4) **Role of the stress in the selection of the velocity direction:** An increasing stress increases the trend towards the stream with respect to the trend towards the target.

(5) **Role of the density in the selection of the velocity direction:** An increasing local density increases the trend towards less congested areas with respect to the stress-weighted trends towards the stream and the target.

(6) **Role of the distance from the wall in the selection of the velocity direction:** The distance from the walls only enters into play if the walking direction encounters a wall. Then the distance contributes to weight the velocity direction selected in unbounded domains with respect to the direction by which walkers on the wall, met along the velocity direction, would move towards the target.

(7) **Adaptation of the speed to the density conditions:** Once the velocity direction has been selected, the dynamics of the speed depends on the difference of the local density in the new direction with respect to the direction before the change. Namely, lower densities increase the speed, while higher densities decrease it. This dynamics is enhanced by the quality of the venue.

Let us now rapidly enlighten the technical differences to develop the modeling approach at each scale:

- The local density is evaluated over the pedestrian's visibility domain with some technical differences at each scale. For *individual-based models*, it is computed as the average of discrete quantities, while for *kinetic models* as integral of the probability distribution function; finally, for *hydrodynamical models* the local density is directly a dependent variable.
- Our minimal model includes only two parameters, namely, the **quality of the venue** α, which affects both components of the dynamics, and the **level of stress** β, which affects the attraction towards the stream with respect to the trend towards the target. Both parameters can have an important influence on the overall dynamics. Therefore, a research perspective consists in investigating their role in the overall dynamics and pattern formation.
- The multiscale vision allows to account for crowd dynamics in venues made of interconnected areas, where the selection of the modeling scale can be differ-

ently related in each area due to their specific features. However, this is not the final step as applications might require either simplified models aiming at reducing the computational complexity or advanced models with the ability of accounting for additional important features of human crowds, like for instance the propagation of emotional states.

5. On the propagation of emotional states

The modeling approach presented in Section 4 is based on the rationale and related mathematical frameworks proposed in Section 3. However, a key problem has been postponed until now, namely, the modeling of the propagation of emotional states which needs to go beyond a dynamics induced by an homogeneously distributed psychological state, like for instance stress conditions up to real panicking which propagate in space, where patterns of high concentration can appear [40].

Contributions of mathematical modeling to this specific dynamics are still limited and almost confined to the kinetic theory approach. A systematic analysis of this challenging topic has been initiated in [16,51] in the case of one dimensional motion. The contagion dynamics is modeled by a consensus interaction somehow analogous the the BGK model of the Boltzmann equation, see for instance [23]. However, the modeling of contagion is not simply a dynamics of consensus towards a commonly shared emotional state, but it should account for communications by vocal or visual signs of walkers who transfer the emotional state across the crowd.

Therefore, it is a problem of collective learning [21,20] which is distributed in space and which can induce significant modifications in the overall self-organization, and hence on the collective dynamics. A modeling approach, accounting at least partially for these features, has been developed in [13] based on the mathematical tools of the kinetic theory for active particles [5]. It essentially consists in introducing in the microscopic state an additional variable accounting for the level of stress, and inserting a social dynamics for such a variable so that space patterns of the emotional state can be studied.

It can be shown how the achievements of [13] can be extended to lower and higher scales. It is not going to be a straightforward generalizations as it needs additional sharp models of interactions. Bearing all these reasonings in mind, let us consider the simple case of a system constituted by one functional subsystem only. The rationale, following the results in [13], can be summarized as follows:

- The mechanical state of the a-particles is defined by the position $\boldsymbol{x}$ and the velocity $\boldsymbol{v} = \{v, \theta\}$, while the emotional state is modeled by a variable which is referred to as activity and is assumed to take values in the domain $[0, 1]$.
- Interactions are supposed not only to modify the mechanical variables, but also the activity which, in turn, may affect the mechanical dynamics. Indeed, different behaviors induce different interactions and, in turn, different pedestrians' trajectories.

- The interaction rate and the dynamics by which walkers modify their velocity direction are modeled according to the same assumptions presented in Section 3.
- The modeling of the transition probability density is based on the assumption that interactions trigger a decision process which comprises the following steps: (1) Exchange of the stress state; (2) Selection of the walking direction; (3) Selection of the walking speed. Decisions are supposed to be sequential and dependent on the local flow conditions being modeled by a transition probability density which factorizes as follows:

$$\begin{aligned}&\mathcal{A}[\rho, \boldsymbol{\xi}](\boldsymbol{v}_* \to \boldsymbol{v}, u_* \to u | \boldsymbol{x}, \boldsymbol{v}_*, u_*, u^*; \alpha, \beta, \varepsilon) = \\ &\qquad \mathcal{A}^v[\rho, \boldsymbol{\xi}](\boldsymbol{v}_* \to \boldsymbol{v} | \boldsymbol{x}, \boldsymbol{v}_*; \alpha, \beta)\, \mathcal{A}^u(u_* \to u | u_*, u^*; \varepsilon). \qquad (5.1)\end{aligned}$$

- The dynamics of the velocity is given by Eq. (3.11) while the emotional state is supposed to spread through the crowd based on the following model of transition probability density:

$$\begin{cases} u^* > u_* \,:\, \mathcal{A}^u(u_* \to u | u_*, u^*; \varepsilon) = \delta\big(u - u_* - \varepsilon(u^* - u_*)(1 - u_*)\big), \\ \\ u^* \leq u_* \,:\, \mathcal{A}^u(u_* \to u | u_*, u^*; \varepsilon) = \delta\big(u - u_*\big), \end{cases} \qquad (5.2)$$

  where ε is a parameter that measures the tendency of pedestrians to modify their emotional state.

6. Critical analysis

In this paper, a multiscale vision has been proposed based on the concept that crowd models at the micro/meso/macro-scale should be derived referring to mathematical structures specific of each scale and by implementing models of interactions which can be obtained by a rationale commonly shared at all scales. The aim of this approach has been the design of new tools towards the modeling and simulation of human crowds in complex venues.

An additional problem, to be taken into account within the framework of a multiscale vision, is the derivation, by averaging or perturbation methods, of models at the macroscopic scale from the underlying description delivered by kinetic theory models. Averaging methods are often developed by a mean field approximation inspired to the celebrated Hilbert problem [38] which has been revisited in [18,19] focusing on the dynamics of multicellular systems previously treated in [7].

The development of these methods requires tackling some nontrivial difficulties, for instance the need of averaging over the sensitivity-visibility domains in the former method or identifying an equilibrium solution to be used as basis for initiate the perturbation method. Some results have been obtained in [4] in unbounded domains, while a possible speed equilibrium solution can be obtained as suggested in [12]. However, a detailed study of this problem for the dynamics in presence of boundaries is still missing. Nevertheless, the approach here proposed can, at least, relate precisely models at each scale guiding the aforementioned derivation.

Finally, let us mention that the approach proposed in this paper can be quite naturally extended to the derivation of models of swarm dynamics, namely, to a research topic promoted by the celebrated Cucker-Smale model [27]. An important contribution to derive kinetic models from the underlying description at the microscopic scale has been given in [34], subsequently revisited by various authors as reported in the survey [2], while the derivation of kinetic models by accounting for models of interactions in the visibility domain has been proposed in [15] and in [14]. The topic which has been briefly outlined above is here addressed to possibly interested readers to be taken as a research perspective. Looking for hyperbolic limits (see [46] as an example) is an objective consistent with the multiscale vision developed in our paper.

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